Int. J. Multiphase Flow, Vol. 4, pp. 511–521. Pergamon/Elsevier, 1978. Printed in Great Britain

ANALYSIS OF A HORIZONTAL SOLID-LIQUID PIPE FLOW BY A CROSS CORRELATION METHOD

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(*Received* 3 *February* 1977, *in revised form* 6 *June* 1978)

Abstract—The transit time distribution has been demonstrated to be a most useful measure of the behaviour of two phases mixtures but the measurement of it with tracers is difficult. The authors have tested a method based on the correlation of fluctuations in the mixture concentration as measured by gamma ray density meters at two sections. This method is shown to give accurate results and allows continuous monitoring of conditions in a pipeline. It is shown how such monitoring can allow the onset of critical conditions such as deposition or the formation of a mobile bed to be detected. This approach can lead to non-dimensional analysis and therefore prediction of mixture behaviour from laboratory tests.

INTRODUCTION

The flow characteristics of heterogeneous solid–liquid mixtures in pipelines depend on many different parameters. It is difficult both to measure these locally and to estimate their interactions.

Fundamental research in this field has been limited, although numerous studies of a global nature have been carried out for industrial applications since 1950.

1. HEAD LOSS AND CRITICAL VELOCITY STUDIES

Head loss measurements are easily undertaken on a pilot plant and allow an estimation of the energy required for transport. Knowledge of the critical velocity allows the prediction of the onset of abnormal behaviour such as deposition, saltation or moving bed.

These studies, of a so called global nature, give little indication on the behaviour of the transported solids, and, as the scaling laws are not well known, their results cannot be extrapolated with confidence.

2. TRANSIT TIME DISTRIBUTION STUDIES

Since the introduction of radioactive tracers, it has become possible to analyse in greater detail the behaviour of mixtures by determining the transit time distribution (TTD).

The TTD is defined, for a given length of pipe S_1S_2, as the density of probability that a small disturbance entering S_1 at time t_0 will leave at S_2 at time t_i. In fluid–solid flow, this small disturbance can be a marked particle.

It describes very generally the movement of all particles and allows examination of the phenomenon liable to occur between S_1 and S_2.

The mean transit time $\bar{t}$ may be calculated from:

$$\bar{t} = \int_0^\infty t \cdot p(t)\,\mathrm{d}t \qquad [1]$$

where: $p(t)$ is the density of probability with

$$\int_0^\infty p(t)\,\mathrm{d}t = 1.$$

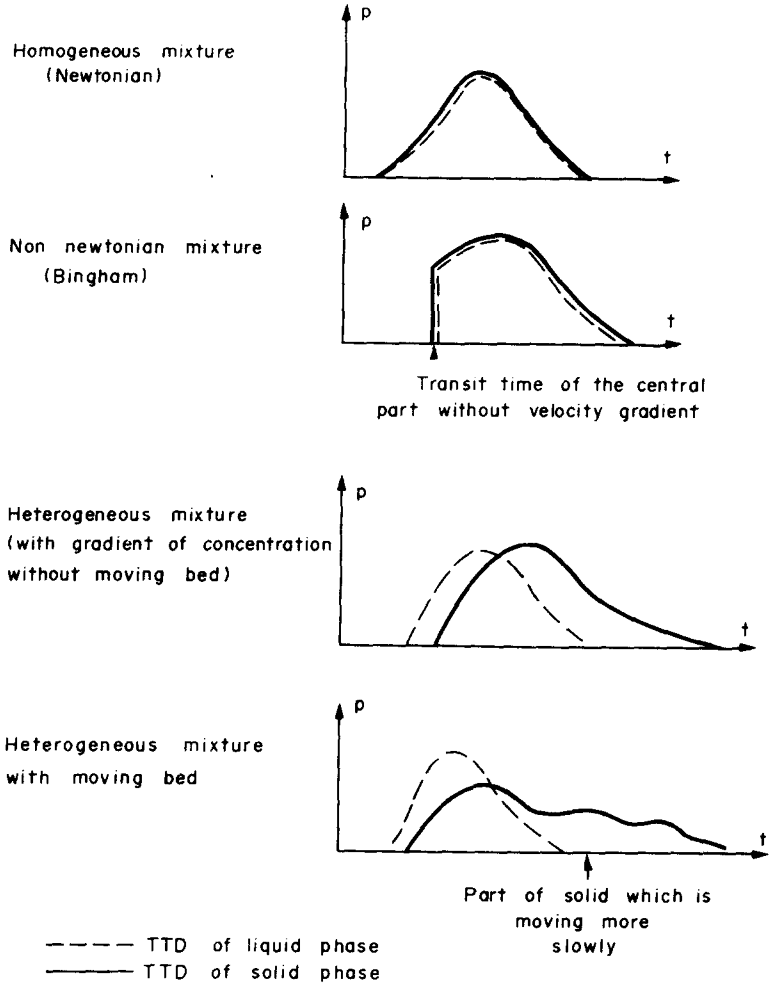

Figure 1. Shape of different TTD.

The shape of the TTD also characterises the mode of transport (figure 1). A homogeneous mixture is characterised by an identity between the TTD of the solid and liquid phases (without interphase relative velocity). For non Newtonian behaviour, as in a Bingham fluid, the motion of the central part, where there is no velocity gradient, will be as shown.

In the case of a heterogeneous mixture, the breadth of the TTD increases with the gradient of concentration. In a mixture with a broad size distribution, the presence of a moving bed with large difference between the velocities of the particles in the moving bed and the particles in suspension gives a TTD with many peaks, displaced with respect to the TTD of the liquid phase.

Once the TTD of a mixture is known, it is possible to fit a mathematical model of convection-diffusion as:

$$\frac{\partial C}{\partial t} + u^i \frac{\partial C}{\partial x^i} = D^i \frac{\partial^2 C}{\partial (x^i)^2}. \qquad [2]$$

With: C = concentration;
t = time;
x^i = space coordinates($i = 1, 2, 3$);
u = velocity;
D = dispersion tensor.

or a stochastic model in which the different momenta can be also related to the behaviour of the mixture.

3. MEASUREMENT OF TTD USING TRACERS

To obtain the TTD of one phase, a representive sample must be marked, and the behaviour of this sample must be typical of that of the whole phase. Reference is made to studies on the subject in conditions of good mixing (Courtois 1964; Alquier 1970; Alquier *et al.* 1970; Max 1972; Mesch *et al.* 1974; Beck *et al.* 1974; Kakka 1974; Guizerix & Margrita 1976).

Instantaneous injection of a tracer is, of course, physically impossible. At each point M of the section S_1 a number of particles must be marked (or injected) in proportion to the solid flux though ds, an elemental section centred on M.

Two methods are commonly used: that of inverse convolution and single particle injection.

3.1. *Inversion convolution*

A sample of the solid phase is injected well enough upstream of section S_1, to ensure complete mixing before S_1 is reached. The temporal distributions of the passage of tracers at sections S_1 and S_2 are, by a process of inverse convolution, used to calculate the TTD.

The drawback of this method is that, as the injection point is far upstream of S_1, the two temporal distributions are flat and therefore the inverse convolution is liable to significant error.

3.2. *Single particle*

In the second method, single particles are injected at S_1 and the transit time measured. The experiment is repeated to obtain a histogram of values for a single particle size, and repeated again for the range of particle sizes in the mixture. The histograms are then weighted according to the particle size distribution of the transported solid to produce a representative TTD.

This method is very slow and is based on the assumption that the flow regime is stationary.

In addition to the drawbacks mentioned for the above two methods, it is seldom possible, for safety reasons, to have the radioactive tracer continuously available.

A general disadvantage of the use of tracers is the need to penetrate the flow with the inevitable risk of causing disturbances to the flow regime. To avoid these problems, the authors have used another method to obtain the TTD based on correlation of the fluctuations of mass.

4. CORRELATION METHOD

In this method, the fluctuations of mass measured at S_1 and S_2 are correlated, and it can be shown that the correlation function is related to the TTD. This technique has been done under contract with the D.G.R.S.T.†

4.1 *Experimental technique*

The fluctuations of flux passing sections S_1 and S_2 are measured by meters which operate on the principle that the absorption of gamma rays depends on the mass of material traversed. These meters have a response time of less than 1 ms and produce an electrical output signal (see figure 2).

Note the term "section" here is taken as equivalent to the volume represented by the product of the area of the gamma ray (3.14 cm^2) and the diameter of the pipeline ($D = 20$ cm).

4.2. *Mathematical procedures*

The five following definitions are used in the analysis:

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†French Général Direction of Scientific and Technical Research.

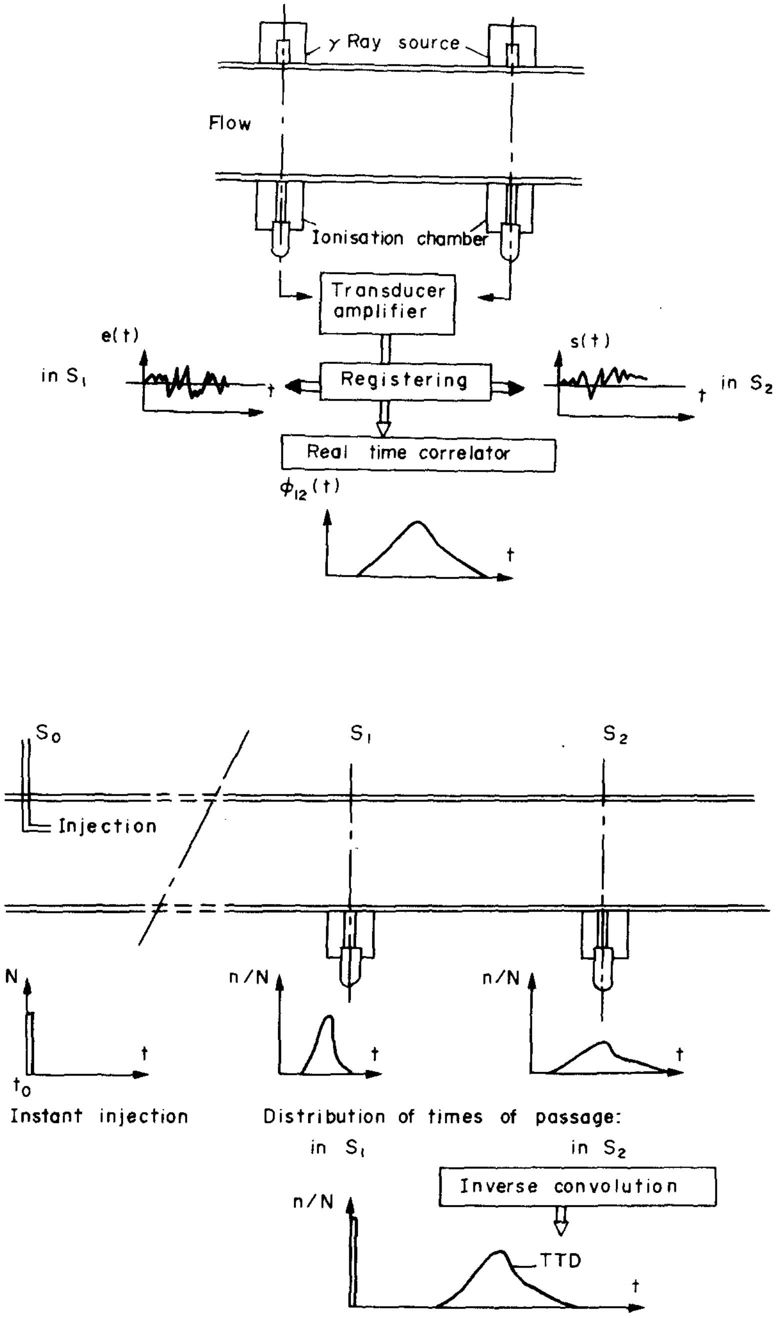


Figure 2. a. Fluctuation crosscorrelation. b. Tracers use.

Dirac impulse (***Delta function***) defined by $e(t) = \infty$ at $t = 0$ and $e(t) = 0$ at $t \neq 0$ with:

$$\int_0^\infty e(t)\,\mathrm{d}t = 1 \qquad [3]$$

Convolution is the transformation of an input signal $e(t)$ by a system S into an output signal $s(t)$ where:

$$s(t) = \int_0^\infty e(u)h(t-u)\,\mathrm{d}u \qquad [4]$$

which may be written symbolically as:

$$s(t) = e(t) * h(t). \tag{5}$$

Note that $h(t)$ is the system response to a Dirac impulse.
Inverse convolution is the inverse of the above, allowing the response to a delta function to be calculated from:

$$h(t) = s(t) \overset{*}{\underset{*}{-}} e(t) \tag{6}$$

The cross correlation function. If $e(t)$ and $s(t)$ are the input and output signals resulting from fluctuations of one parameter, then the cross correlation function may be written:

$$\phi_{12}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T e(t)s(t-\tau)\,dt \tag{7}$$

The auto-correlation function is defined as:

$$\phi_{11}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T e(t)e(t-\tau)\,dt \tag{8}$$

hence

$$\phi_{12}(\tau) = \int_0^T h(t)\phi_{11}(\tau - t)\,dt. \tag{9}$$

Thus, the function ϕ_{12} relates the output signal to the memory of the input signal (Max 1972).

The theory uses the fact that, as the auto-correlation function tends to a Dirac impulse, the cross-correlation function tends to the impulse response $h(t)$ of the system.

i.e. $$\phi_{11}(\tau) \to \delta(\tau),$$

then $$\phi_{12}(\tau) \to h(\tau).$$

In this analysis, the signals $e(t)$ and $s(t)$ are the fluctuations of absorption around the mean value, which corresponds to the mean value of mass in the "sections", $\bar{m}$.

$e(t)$ or $s(t)$ correspond to $m'(t)$ with:

$$m(t) = \bar{m} + m'(t). \tag{10}$$

4.3. *Determination of the TTD*

The length of pipeline (S_1S_2) is treated as a closed system S which is assumed to be linear.

A Dirac impulse cannot be physically measured but the correlators can give the auto-correlation function ϕ_{11} which is, in the case of "white noise", equivalent to a Dirac impulse.

If the auto-correlation function had been too wide to be considered as a delta function, the impulse response could have been obtained by inverse convolution:

$$h(t) = \phi_{12}(t) \overset{*}{\underset{*}{-}} \phi_{11}(t). \tag{11}$$

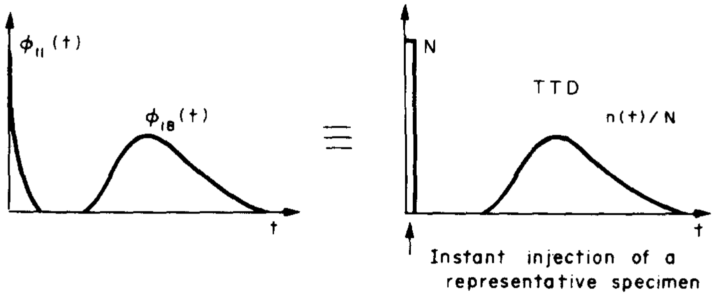


Figure 3. Instant injection of a representative specimen.

Physically, a small disturbance in mixture flow may be the passage of one more or one less particle in S_1 and S_2, around the mean value. If the system is linear and if the process is ergodic, we can write:

$\delta(t)$ = superposition at the same instant t of small disturbances (s.d)

$$\delta(t) = \sum (\text{s.d}). \tag{12}$$

The impulse response $h(t)$ will then be the TTD of these small disturbances:

$$h(t) = \text{TTD (s.d)} = \text{TTD (particles)} \tag{13}$$

assuming the flow is free of mass waves, because the transit of the disturbances must depend only on the transit of the particles.

Finally, if $\phi_{11}(t) \simeq \delta(t)$:

$$\text{TTD (particles)} = \text{TTD(s.d)} = h(t) = \phi_{12}(t) \tag{14}$$

or, exactly:

$$\text{TTD(particles)} = h(t) = \phi_{12}(t) \overset{*}{\underset{*}{-}} \phi_{11}(t). \tag{15}$$

5. RESULTS

The results of the mass correlation method were compared with the TTD measured by tracers for the same mixtures. The identity it was sought to verify may be shown schematically as figure 3.

5.1. *Transport of fine sand*

Figure 4 shows the auto-correlation function of the mass fluctuations for fine sand and it may be seen that this function can be considered as a unit impulse. The inverse convolution was therefore unnecessary, the differences between:

$$h(t) = \phi_{12}(t) \overset{*}{\underset{*}{-}} \phi_{11}(t) \text{ and } h(t) = \phi_{12}(t)$$

being insignificant.

Figure 5 shows the cross-correlation function of the mass fluctuations and may be compared with figure 6, the distribution of passage times of particles which are marked with Na^{24} and injected at S_0, for upstream of S_1. The mass concentration of sand was 15% and the volume fraction was 6%.

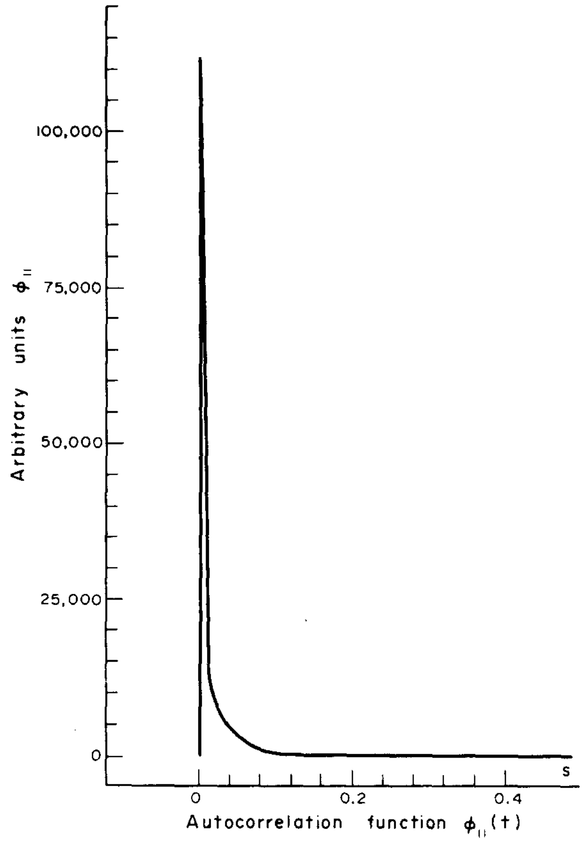
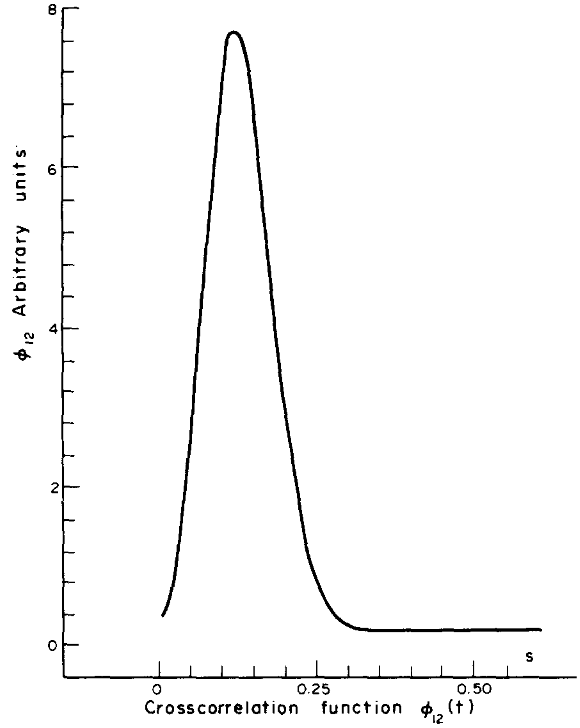


Figure 4. Autocorrelation function.



Mean time = 0.12s - S_1S_2 = 0.5m Mean velocity $\bar{V}$ = 4.16m/s

Figure 5. Cross correlation function.

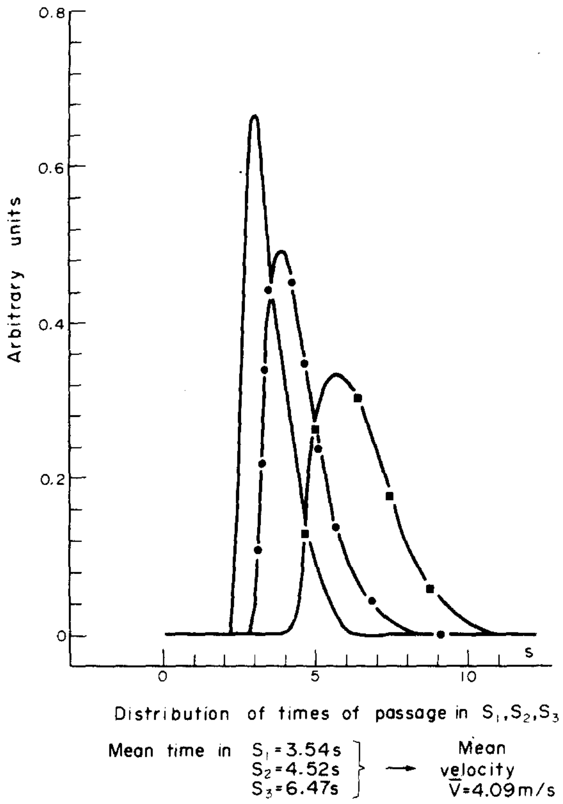


Figure 6. Distribution of times of passage in S_1, S_2, S_3.

Preliminary inspection of the mean passage times between S_1 and S_2. and S_2 and S_3 compared with these of the correlation function, gives the results shown in table 1 below:

Table 1.

Method	Distances between S_1 and S_2 (m)	Mean transit time (s)	Mean transit time per m (s)	Mean velocity of the dispersed phase (m/s)
Correlation	0.5	0.12	0.240	4.16
Tracer	S_1 S_2 4	0.98	0.245	4.09
	S_2 S_3 8	1.95	0.244	4.09

These results correspond to within 2%, which confirm the hypothesis in respect of the first moment of the TTD (higher moments, of variance and skew, have not been included in the study).

5.2. *Transport of gravel*

The experiment was repeated with gravel, with a mean diameter of 5 mm., the results being shown in figure 7. The single particle injection technique was used to build up the histogram, based on a total of 693 particles releases. The increased size of particles allows the spacing of the section S_1 and S_2 to be increased to 2 m since the signal/noise ratio is higher.

Despite the limited number of particles injected, there is a good comparison between the

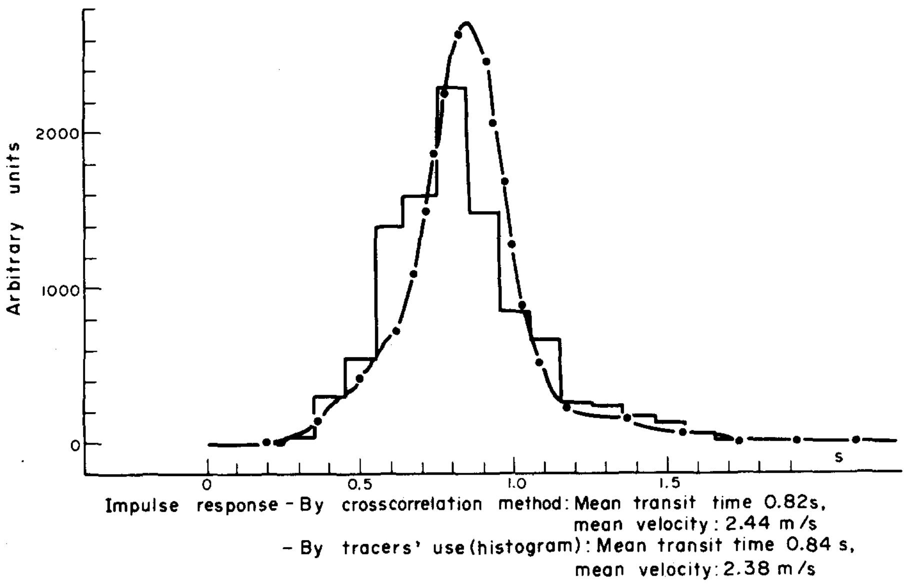


Figure 7. Impulse response.

mean transit time of the particles given by the two methods, as shown in table 2. The difference of 3% is well within the range of experimental error.

Table 2.

Method	Distance between S_1 and S_2 (m)	Mean transit time (s)	Mean transit time per m. (s)	Mean discharge velocities of the gravel (m/s)
Correlation	2	0.82	0.41	2.44
Tracer	2	0.84	0.42	2.38

5.3. *Double convolution operation*

The short length of pipeline S_1S_2 over which the cross-correlation function is measured raises doubts as the validity of the results when applied over longer distances, as dispersion effects will not be necessarily representative of overall behaviour of the mixture.

To test this, therefore, a comparison was made between the cross-correlation function measured for a 2 m length, and a double convolution of the impulse response measured from 0.5 m length. The results, shown in figure 8, indicate that the transit times predicted are practically identical at 0.84 s.

6. CONCLUSION

The correlation method of measuring TTD allows an accurate prediction to the mode of the behaviour of the mixture in a given pipeline without the problems associated with radioactive tracers (transport, security, half life, cost....).

The use of gamma ray meters raises the possibility of measuring the conveyed concentration once a calibration has been made. By plotting the average concentration over a period of time as in figure 9, the appearance of a deposit or a mobile bed can be detected and the flow adjusted accordingly.

By classical methods of liquid tracing the mean slip between the two phases composing the mixture can also be measured (Alquier *et al.* 1970; Guizerix & Margrita 1976; Santos-Cottin 1976).

Finally, it is suggested that flows with the same impulse responses will probably behave in a

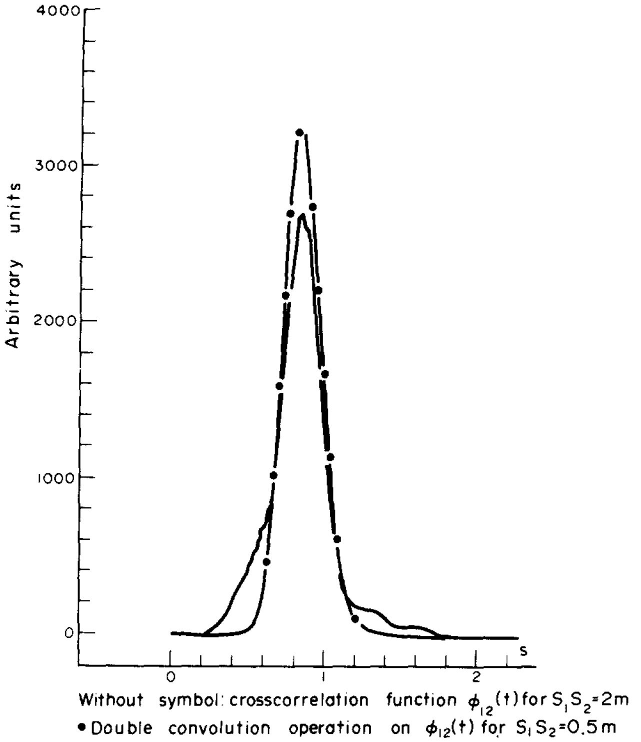


Figure 8. Crosscorrelation function and double convolution operation.

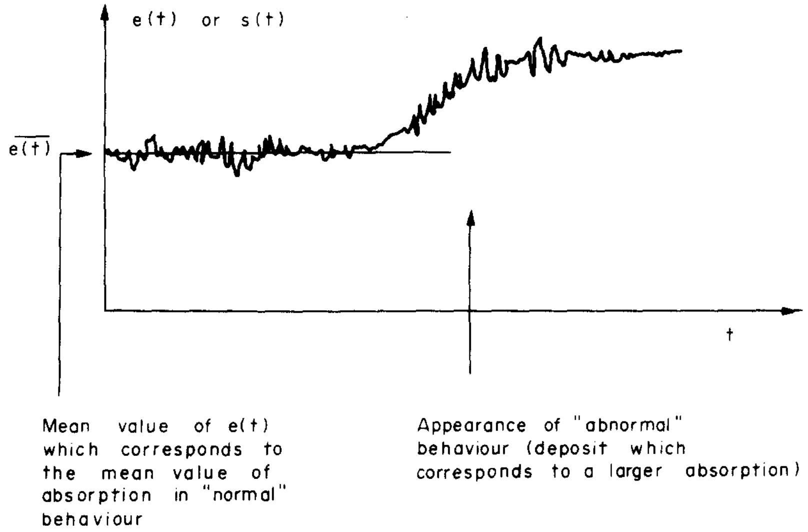


Figure 9. Illustration of the signals obtained in correlation method.

similar manner and show the same characteristics (identical non dimensional concentration profiles and velocity profiles).

Acknowledgements—The authors are most grateful to Pr. Gruat and Dr. Guizerix for their encouragement and support throughout this project.

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